Exercise-set 3, Algorithms and Networks 2012/2013

Please hand in the solutions on paper to Hans Bodlaender on or before the deadline, i.e., Monday, October 8, 2012.

You can write either in English or in Dutch. Your work should be well phrased, readable, understandable, and of course, correct.

Work that is hard to read or very messy will be graded with a 0. Do not forget your name and student number on your work.

Bonus questions are for extra points.

1. Easy question. 1 point What is the triangle inequality for TSP? Can you give an example of an application for TSP where the triangle inequality would not hold?

2. Model and heuristics for a generalisation of TSP. 3 points A company has the following problem. Five lorries must carry goods to customers. In the morning the lorries are at a depot; in the evening, the lorries must return to the depot. We are given the distances from the depot to the set of addresses where the customers are, and the distances between the addresses. These fulfil the triangle inequality. We have a good for each customer, thus each customer must be visited by one lorry. We want a set of routes for the lorries, such that all goods are delivered, but also each lorry can carry at most D goods. Possibly, lorries can return to the depot to collect new goods.

(a) Give a mathematical model for the problem to find routes that fulfil the criteria, such that the maximum time that a lorry travels in minimised.

(b) Propose one or more construction heuristics for the problem.

(c) Propose one or more improvement heuristics for the problem.

4. TSP: modelling. 2 points There is a metal plate, where three different lasers should beam at at different spots. We have three sets of positions on the plate: the first gives the positions where the first laser should beam, etc. The metal plate can move, while the lasers are immobile. The costs of moving the plate from position (x_1, y_1) to position $(x_{2,2})$ is $|x_1 - x_2| + |y_1 - y_2|$.

Can we model this problem as an instance of TSP? If so, show how. Does the triangle inequality hold? If not, which heuristic would you use to solve this problem?

5. Arbitrage. 3 points Suppose we are given n currencies $c_1, c_2, \ldots, c_n$. For each pair of currencies c_i, c_j, $i, j \in \{1, 2, \ldots, n\}$, we are given the conversion rate: a number $R(i, j)$ is given, such that we can convert α units of currency c_i to $R(i, j) \cdot \alpha$ units of currency c_j.

For instance, if c_i is euro, and c_j is US dollar, and $R(i,j) = 1.1$, then we can change 1 euro to 1.1 US dollar.

Some traders are investigating conversion rates to see if there is a 'loop' such that we can earn money (in the original currency). This is called arbitrage.

So, we look for a sequence of currencies $c_{i_1}, \ldots, c_{i_r}$, such that if we change some amount in currency c_{i_1} to currency c_{i_2}, then the just obtained amount from currency c_{i_2} to currency c_{i_3}, etc., and finally, what we got in currency c_{i_r} to currency c_{i_1}, we have a larger amount than with what we started.

- Explain that we can model this as finding a cycle in a graph with some property. What is the property we look for?
- Give an efficient algorithm to test if such a cycle exists. Explain why your algorithm is correct. Analyse the running time of your algorithm. (Your algorithm should use in any case polynomial time. What is the best bound (as a function of n, using O-notation) that you are able to get?)

6. Difficult question. 1 point Suppose we have an instance of TSP that is not symmetric but still fulfils the triangle inequality. (Consider a postman on a bike in a mountainous area.)

Suppose that for every two cities i and j, we have that $d(i,j) \leq 2 * d(j,i)$. (And, of course, $d(j,i) \leq 2 * d(i,j)$.

Give an approximation algorithm that runs in polynomial time and has a performance ratio bounded by a constant.

Bonus points (maximum 3) if you have very good performance ratios.