Exercises Algorithms and Networks – Set 5 - 2012-2013

Please note:

- Hand in your work on paper. In special circumstances, you can email your work to H.L.Bodlaender@uu.nl. In such a case, have [AN5] in the header.
- Write clear and neat. Messy work can be graded with a 0.
- Write always your name and student number.
- Deadline for this set: Monday October 22, 2012.

Bonus questions may be difficult, are optional, and give extra points. The maximum note you can get for this set is 13.

Do not forget your student number. Either staple your solutions, or write on the first paper the number of pages you hand in and write on each paper your name.

1. First questions. 2 points

(a) Suppose we have a bipartite graph $G = (V, E)$ with for each edge a value $v(e)$. Values are positive integers. Explain how we can solve the problem to find a perfect matching of maximum total value, if it exists, with help of minimum cost flow algorithms. (Hint: draw a small example.)

(b) Suppose that we want to find a matching of maximum total value, i.e., we allow that some vertices are not matched, is it possible to find such with just one run of a minimum cost flow algorithm?

2. Phasing out capital equipment. 3 points

Exercise 9.6 from Network Flows, Ahuja, Magnanti, Orlin.

A shipping company wants to phase out a fleet of (homogeneous) general cargo ships over a period of p years. Its objective is to maximize its cash assets at the end of the p years by considering the possibility to prematurely selling ships and temporary replacing them by ships they rent.

The company faces a non-increasing demand for ships: every year, the number of ships needed is at most the number needed the previous year. Let $d(i)$ be the number of ships needed in year i. The company can possibly have more than $d(i)$ ships in year i, but may not have less than these.

If a ship is sold in year i, this yields s_i euros.

Each ship the company has in year i gives r_i euros as profit.

If the company has less than $d(i)$ ships in year i, it must hire additional ships. This costs h_i in year i. Assume that $h_i > r_i$ for each i.

The company wants to meet its commitments (have sufficient ships either owned or rented each year), and have as much cash assets as possible at the end of the pth year.

Formulate this problem as a minimum cost flow problem.

3. Maximum size simple b-matching. 5 points

Consider an undirected graph $G = (V, E)$ with a function $b : V \to \mathbf{Z}^+$. A simple b-matching is a set of edges $F \subseteq E$, such that every vertex v is incident to at most $b(v)$ edges in F.

We want to find a maximum size simple b-matching of maximum size, given G and b. In this exercise, we show that we can find such a matching in polynomial time. This is done by solving the maximum matching matching problem on another graph.

Let $H = (V', E')$ be the graph, obtained from G in the following way.

- For each vertex $v \in V$, we take $b(v)$ vertices $v_1, \ldots, v_{b(v)}$ in V'.
- For each edge $e = \{v, w\}$ in E, we take two vertices $x_{e,v}$ and $x_{e,w}$ in V'.
- For each edge $\{v, w\}$ in E, we take an edge $\{x_{e,v}, x_{e,w}\}$ in E'.
- For each edge $\{v, w\}$ in E, and for each i, $1 \leq i \leq b(v)$, and for each j, $1 \leq j \leq b(w)$, we take the edges $\{v_i, x_{e,v}\}$ and $\{w_j, x_{e,w}\}$ in E'.

1. (Optional.) Draw a small example graph G with weight function b, and show how the graph H resulting from this construction looks like for this example.
2. Show that for all integers k: if G has a simple b-matching with k edges, then H has a simple matching with $k + |E|$ edges.
3. Show that for all integers k, if H has a simple matching with $k + |E|$ edges, then G has a simple b-matching with k edges. (Hint: analyse what happens in the edges that replaces one edge in G.)
4. Let for all $v \in V$, $b'(v) = \min\{b(v), \text{degree}(v)\}$. Explain that the maximum size of a simple b-matching in G equals the maximum size of a simple b'-matching in G.
5. Argue that the problem to find a simple b-matching of maximum size can be solved in polynomial time.
6. **Bonus question:** A k-flower in an undirected graph is a set of cycles such that there is one vertex v, such that all cycles use v, but have no other vertices in common. (I.e., for each pair of cycles in the set, v is the only vertex on both cycles.) Show that we can decide in polynomial time if G has a k-flower. (This is used as a step in a *kernelisation algorithm for feedback vertex set*.) Hint: find a 'perfect' b-matching in a modification of G.

Bonus question

In a (two-dimensional) computer game, a spacecraft must fly through a narrow corridor. In the corridor are beacons. Each beacon has a range of 1 distance unit. White the beacons should be friendly, due to a programming malfunction, they will fire at the spacecraft and destroy it when the spacecraft enters the range.

Now, the spacecraft can shoot at the beacons and thus destroy them. However, when doing so, the player gets -1 point (as the beacons were friendly).

The spacecraft is at the beginning of the corridor, and must reach the end. So, we must destroy beacons such that a clear path from the beginning to the end is formed. We assume that the size of the spacecraft can be neglected. The question is: what is the minimum number of beacons the spacecraft must destroy.

Give an algorithm that solves this problem in polynomial time and argue that it is correct.

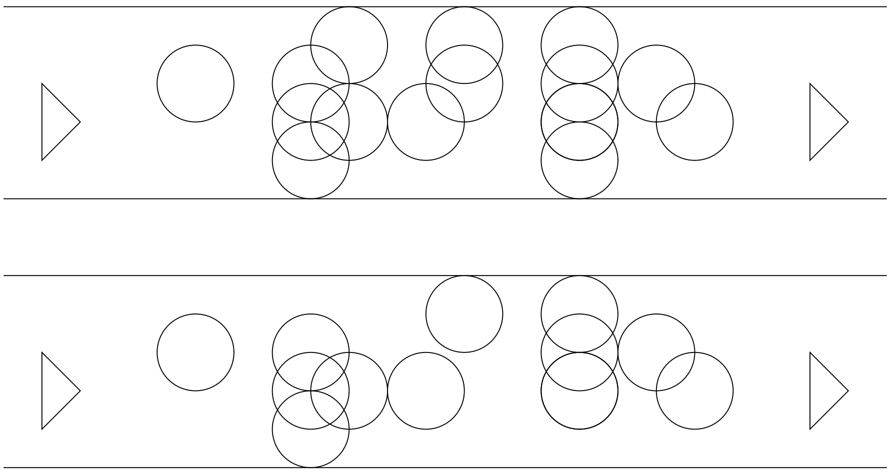

Figure 1: Example: before and after destroying beacons