Exercises Algorithms and Networks – Set 8 - fall 2012

Deadline: Monday, November 12, 2012.

You can either:

- Hand in these exercises on paper by Hans Bodlaender or in his mailbox on the 5th floor of the BBL, or
- Send a pdf-file as attachment by email to H.L.Bodlaender@uu.nl with [AN8] in the header of the email.

Grading is as follows. The exercise set has five parts: 1, 2, 3, 4(i) and (ii), and 4(iii). If you have one part correct, you score a 3; if you have two parts correct, you score a 5; if you have three parts correct, you score an 7.5; with four parts, you score a 9, and a total correct set scores a 10. The bonus question can lift your note for this set to a note to something above 10, never more than 13. (2 points for a good solution, up to 3 points for something very clever.) (Correctness also includes clear and neat work; partial scores are also possible.)

1. Small degree graphs

Show that the following problem belongs to FPT.

> **Given:** Graph $G = (V, E)$ such that each vertex in G has degree at most three, integer k.
> **Question:** Does G have a dominating set of size at most k?
> **Parameter:** k

(*The question has a simple answer, but also complicated solutions can be correct.*)

2. Independent dominating set on trees

A set of vertices $W \subseteq V$ is an *independent dominating set* in a graph $G = (V, E)$, if

1. for all $v, w \in W$: $\{v, w\} \notin E$ (i.e., W is an independent set), **and**
2. for all $v \in V - W$: there exists a $w \in W$ with $\{v, w\} \in E$ (i.e., W is a dominating set).

Give a linear time algorithm that computes the minimum size of an independent dominating set of a given **tree**. Explain your algorithm also in words.

Hint: Use dynamic programming.

3. Treewidth of series parallel graphs

A special type of graphs are the series parallel graphs.

Series parallel graphs have two special vertices, their *terminals*. G is a series parallel graph with terminals s, t, if one of the following three cases holds:

- G is a single edge $\{s, t\}$.
- There are series parallel graphs G' with terminals s', t', and G'' with terminals s'', t''. We obtain G by taking the disjoint union of G' and G'' and then identifying t' and s''. (Series-composition.)
- There are series parallel graphs G' with terminals s', t', and G'' with terminals s'', t''. We obtain G by taking the disjoint union of G' and G'' and then identifying s' and s'', and identifying t' and t''. (Parallel-composition.)

Show that series parallel graphs have treewidth at most two.

4: Modulator to single edges

Consider the following parameterized problem:

> Modulator to single edges - parameterized version
> **Given**: Graph $G = (V, E)$, integer $k \geq 0$
> **Question**: Is there a set of vertices $W \subseteq V$ such that the graph obtained by removing all vertices in W consists of only vertices of degree 0 and 1 and $|W| \leq k$?
> **Parameter:** k

I.e., when W is removed, the remainder of the graph are single vertices and edges. Call such a set W a *modulator to single edges*.

And consider its optimization version:

> Modulator to single edges - parameterized version
> **Given**: Graph $G = (V, E)$
> **Question**: Give a minimum size set of vertices $W \subseteq V$ such that the graph obtained by removing all vertices in W consists of only vertices of degree 0 and 1?
> **Parameter:** k

(i) Let v be a vertex of degree at least two. Let w and x be two of the neighbors of v. Let S be a modulator to single edges. Argue that $\{v, w, x\} \cap S \neq \emptyset$.

(ii) Consider the following algorithm.

$S = \emptyset$;
Let $H = G$; *i.e., we copy G to the graph H*;
while H contains at least one vertex of degree at least two **do**
    Take a vertex v of degree at least two in H.
    Take two neighbors w and x of v in H;
    Add v, w, and x to S;
    Remove v, w and x from H, together with all edges with v, w or x as endpoint;
    enddo;
Output S;

Argue that this algorithm is a 3-approximation for the MODULATOR TO SINGLE EDGES - OPTIMIZATION VERSION problem. I.e., the set S that is given as output is at most three times as large as an optimal solution to the problem.

(iii) Give a kernel for MODULATOR TO SINGLE EDGES - PARAMETERIZED VERSION. Analyze its size. (Hint: Compare with vertex cover.)

Bonus question

This question is NOT obligatory. It can give you bonus points.

Show that the MODULATOR TO SINGLE EDGES - PARAMETERIZED VERSION is in FPT with a branching algorithm. Analyse its running time.