Proving Compiler Correctness with Dependent Types

João Paulo Pizani Flor
Wout Elsinghorst

Department of Information and Computing Sciences
Utrecht University

Wednesday 16th April, 2014

Universiteit Utrecht

Table of Contents

Table of contents

Source language, Target language

- Example source code (expression language):

```
Add (Val 1) (Add (Val 1) (Val 3))
```

- Example target code (for a stack machine):

```
PUSH 1 >> PUSH 1 >> PUSH 3 >> ADD >> ADD
```

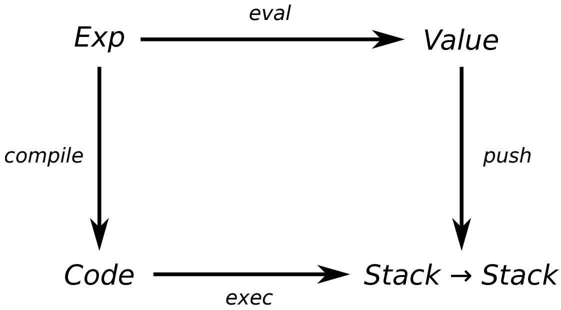
Evaluation, execution

- An **eval** function gives the semantics for the **source** language
  - Denotational semantics
  - Maps terms to values
- An **exec** function gives the semantics for the "**machine**" language
  - For each instruction, an operation to perform on the machine state (stack)

Table of contents

What does "correct" mean?

- Both semantics (before and after compilation) should be "equivalent"
- Compiling then executing must give the same result as direct evaluation



Reference paper

- "A type-correct, stack-safe, provably correct expression compiler in Epigram"
  - James McKinna, Joel Wright
- Basic ideas and proofs, which we extended...

Table of contents

Introduction
Context/Terminology
Compiler correctness
Sharing
Goals
Implementation (code)
Basic correctness
Lifting to sharing setting
Conclusions

Universiteit Utrecht

Extending the source language

- More "realistic" languages have sharing constructs
- We wanted the "simplest possible" extension with sharing behaviour.
- **Chosen extension: if_then_else + sequencing**

```
if c then t else e >> common-suffix
```

- The "naïve" compile function will duplicate the suffix
- Having Bytecode defined as graph (structured graph) instead of tree would solve this problem
  - But proofs would be more complex

Table of contents

What we ideally want

- Have a "smart" graph-based compiler, generating code which uses sharing
- Write the correctness proof only for the "dumb" compiler, have correctness **derived** for the smart version.

Reference paper

- "Proving Correctness of Compilers using Structured Graphs"
    - Patrick Bahr (visiting researcher)

Our project's goals

- Integrating the best of both "reference" papers
- Our contributions:
  - (Simplest possible) language extension showing sharing behaviour.
  - Proof of correctness for the **stack-safe** "naïve" compiler
    - The one that just duplicates code.
  - A way to to lift this **stack-safe** "naïve" correctness proof
    - Into a proof concerning the more **efficient** compiler.

Table of contents

Source

- Source types:

$$\begin{aligned}&\text{data } \mathsf{Ty}_s : \mathsf{Set} \text{ where}\\ &\quad \mathbb{N}_s : \mathsf{Ty}_s\\ &\quad \mathbb{B}_s : \mathsf{Ty}_s\end{aligned}$$

- Source terms (snippet):

$$\begin{aligned}&\text{data } \mathsf{Src} : (t : \mathsf{Ty}_s) \to (z : \mathsf{Size}_s) \to \mathsf{Set} \text{ where}\\ &\quad \mathsf{v}_s \quad : \forall \{t\} \to (v : \{\!\!\{ t \}\!\!\}) \to \mathsf{Src}\ t\ 1\\ &\quad \_+_s\_ \quad : (e_1\ e_2 : \mathsf{Src}\ \mathbb{N}_s\ 1) \to \mathsf{Src}\ \mathbb{N}_s\ 1\end{aligned}$$

- Denotational semantics (snippet):

$$\begin{aligned}&[\![\_]\!] : \{t : \mathsf{Ty}_s\}\ \{z : \mathsf{Size}_s\} \to (e : \mathsf{Src}\ t\ z) \to \mathsf{Vec}\ \{\!\!\{ t \}\!\!\}\ z\\ &[\![\ \mathsf{v}_s\ v\ ]\!] \quad = [\ v\ ]\\ &[\![\ e_1 +_s e_2\ ]\!] \quad = [\ [\![\ e_1\ ]\!]' + [\![\ e_2\ ]\!]'\ ]\end{aligned}$$

Bytecode

- Typed stack:

```
StackType : Set
StackType = List Ty_s


data Stack : StackType → Set where
    ε      : Stack []
    _∇_    : ∀ {t s'} → { t } → Stack s' → Stack (t :: s')
```

- Typed bytecode (snippet):

```
data Bytecode : StackType → StackType → Set where
    SKIP : ∀ {s}      → Bytecode s s
    PUSH : ∀ {t s}    → (x : { t }) → Bytecode s (t :: s)
    ADD  : ∀ {s}      → Bytecode (ℕ_s :: ℕ_s :: s) (ℕ_s :: s)
```

Compiler correctness

$$
\begin{aligned}
&\mathsf{compile} : \forall\ \{t\ z\ s\} \to \mathsf{Src}\ t\ z \to \mathsf{Bytecode}\ s\ (\mathsf{replicate}\ z\ t +\!\!+\ s)\\
&\mathsf{compile}\ (\mathsf{v}_s\ x) \qquad\quad = \mathsf{PUSH}\ x\\
&\mathsf{compile}\ (e_1 +_s e_2) \quad = \mathsf{compile}\ e_2 \rangle\!\rangle\ \mathsf{compile}\ e_1 \rangle\!\rangle\ \mathsf{ADD}
\end{aligned}
$$

$$
\begin{aligned}
&\texttt{correct : \{t : Ty}_s\texttt{\} \{z : Size}_s\texttt{\} (e : Src t z)}\\
&\qquad \to \texttt{exec (compile e)} \equiv [\![\ \texttt{e}\ ]\!]
\end{aligned}
$$

Table of contents

Tree fixpoints

- Fixed Point for standard Functors

```
data Tree (f : Set → Set) : Set where
  In : f (Tree f) → Tree f
```

- Fixed Point for indexed Functors

```
data HTree
      (f : (StackType → StackType → Set)
         → (StackType → StackType → Set) )
      (ixp : StackType) (ixq : StackType) : Set where
  HTreeIn : f (HTree f) ixp ixq → HTree f ixp ixq
```

Bytecode Tree Representation

$$
\begin{array}{l}
\textsf{data Bytecode} : \textsf{StackType} \to \textsf{StackType} \to \textsf{Set where} \\
\quad \textsf{SKIP} : \forall \{s\} \quad \to \textsf{Bytecode}\ s\ s \\
\quad \textsf{PUSH} : \forall \{t\ s\} \quad \to (x : \{\!\![\ t\ ]\!\!\}) \to \textsf{Bytecode}\ s\ (t :: s) \\
\quad \textsf{ADD} \ : \forall \{s\} \quad \to \textsf{Bytecode}\ (\mathbb{N}_s :: \mathbb{N}_s :: s)\ (\mathbb{N}_s :: s)
\end{array}
$$

$$
\begin{array}{l}
\textsf{data BytecodeF}\ (r : \textsf{StackType} \to \textsf{StackType} \to \textsf{Set}) : \\
\quad (\textsf{StackType} \to \textsf{StackType} \to \textsf{Set})\ \textsf{where} \\
\quad \textsf{SKIP'} : \forall \{s\} \quad \to \textsf{BytecodeF}\ r\ s\ s \\
\quad \textsf{PUSH'} : \forall \{a\ s\} \quad \to (x : \{\!\![\ a\ ]\!\!\}) \to \textsf{BytecodeF}\ r\ s\ (a :: s) \\
\quad \textsf{ADD'} \ : \forall \{s\} \quad \to \textsf{BytecodeF}\ r\ (\mathbb{N}_s :: \mathbb{N}_s :: s)\ (\mathbb{N}_s :: s)
\end{array}
$$

- Bytecode is isomorphic to `HTree BytecodeF`
  - `fromTree` ∘ `toTree` ≡ `id`
  - `toTree` ∘ `fromTree` ≡ `id`

Correctness on Trees

```
compileT : ∀ {t z s} → Src t z
    → HTree BytecodeF s (replicate z t ++ s)
```

```
execT : ∀ {s s'} → HTree BytecodeF s s'
    → Stack s → Stack s'
```

```
correctT : ∀ {t z s'} → (e : Src t z)
           → execT (compileT e) ≡ ⟦ e ⟧
```

- Proof of `correctT` can be derived from `correct`
  - Because ‘Bytecode‘ is structurally the same as ‘HTree BytecodeF‘

Graphs

```
data HGraph .. : ... -> Set where ...
```

- `HGraph` is similar ("includes") `HTree`
  - But with extra constructors to represent *shared subgraphs*
- `Bytecode` is not *exactly* isomorphic to `HGraph BytecodeF`:
  - We have: `fromGraph` $\circ$ `toGraph` $\equiv$ `id`
  - But: `toGraph` $\circ$ `fromGraph` $\neq$ `id`
  - `HGraph` $\rightarrow$ `Bytecode` $\rightarrow$ `HGraph` loses sharing

Bytecode Graph Representation

$$\text{compileG} : \{s : \text{StackType}\} \to \forall\ \{z\ t\} \to \text{Src}\ t\ z$$
$$\to \text{HGraph BytecodeF}\ s\ (\text{replicate}\ z\ t \mathbin{++} s)$$

$$\text{execG} : \forall\ \{s\ s'\} \to \text{HGraph BytecodeF}\ s\ s'$$
$$\to \text{Stack}\ s \to \text{Stack}\ s'$$

```
correctG : ∀ {t z} → (e : Src t z)
         → execG (compileG e) ≡ ⟦ e ⟧
```

Using machinery, we get this proof derived from ‘correctT‘

Achieved

- Agda "framework" for deriving compiler correctness proofs
  - Compilers with **typed source** and **typed target**
  - Given correctness of a "naïve" compiler, derive correctness of "optimized" version
- Correctness proof for an expression language (with sequencing)
  - As "instance" of this framework

Agda limitations we faced

- Strict positivity requirement
  - When defining fixed point type operators
- Totality checker
  - When defining folds
- Type checking with positivity check disabled made debugging hard
  - Stack overflow, memory consumption

Yet to be done

- Sequence clause of "basic" (non-lifted) correctness proof
- Prove a final lemma to complete the lifting (fusion law)

Thank you!

Questions?