Π-Ware: An Embedded Hardware Description Language using Dependent Types

Author: João Paulo Pizani Flor
<joaopizani@uu.nl>

Supervisor: Wouter Swierstra
<w.s.swierstra@uu.nl>

Department of Information and Computing Sciences
Utrecht University

Tuesday 26th August, 2014

Universiteit Utrecht

What is Π-Ware

Universiteit Utrecht

What is Π-Ware

"*Π-Ware* is a Domain-Specific Language (DSL) for hardware, embedded in the dependently-typed *Agda* programming language. It allows for the description, simulation, synthesis and verification of circuits, all in the same language."

Universiteit Utrecht

Background

Universiteit Utrecht

Hardware design is hard(er)

- Strict(er) correctness requirements
  - You can't simply *update* a full-custom chip after production
    - Intel Pentium's `FDIV`
  - Expensive verification / validation
    - Up to 50% of development costs
- Low-level details (more) important
  - Layout / area
  - Power consumption / fault tolerance

Hardware design is growing

- Moore's law will still apply for some time
  - We can keep packing more transistors into same silicon area
- **But** optimizations in CPUs display diminishing returns
  - Thus, more algorithms *directly* in hardware

Universiteit Utrecht

Hardware Description Languages

- All started in the 1980s
- *De facto* industry standards: VHDL and Verilog
- Were intended for *simulation*, not modelling or synthesis
  - *Unsynthesizable* constructs
  - Widely variable tool support

Functional Hardware Description

- A functional program describes a circuit
  - Easier to *reason* about program properties
  - Inherently *parallel* and *stateless* semantics
- Several *functional* HDLs during the 1980s
  - For example, μFP [Sheeran, 1984]
- Later, *embedded* hardware DSLs
  - For example, Lava (Haskell) [Bjesse et al., 1998]

Embedded DSLs for Hardware

- Lava
  - Simulation / Synthesis / Verification
  - Limitations: almost untyped / no *size checks*

```
adder :: (Signal Bool, ([Signal Bool], [Signal Bool]))
      -> ([Signal Bool], Signal Bool)
```

- Others:
  - ForSyDe [Sander and Jantsch, 1999]
  - Hawk [Launchbury et al., 1999], etc.

Dependently-Typed Programming

- Dependent type systems: systems in which types can *depend on values*
- It makes a big difference:
  - More expressivity
  - *Certified programming*
- DTP often touted as "successor" of functional programming
  - Very well-suited for DSLs [Oury and Swierstra, 2008]

Research Question / Methodology

Universiteit Utrecht

Research Question / Methodology

- **Question:**
  - What are the improvements that Dependently-Typed Programming (DTP) can bring to hardware design?
    - Compared to other functional hardware languages
- **Methodology:**
  - Develop a hardware DSL, *embedded* in a dependently-typed language (Agda)
    - Allowing simulation, synthesis and verification

Big picture

Dependently-Typed Programming

- **Disclaimer:** Suspend disbelief in syntax
  - Examples are in *Agda*
  - Syntax similar to Haskell, details further ahead
- Types can depend on values
  - Example:

    $$\texttt{data Vec } (a : \texttt{Set}) : \mathbb{N} \to \texttt{Set where} \ldots$$
  - Compare with Haskell:

```
data List (a :: *) :: * where
```
- Types of arguments can depend on *values of previous arguments*
  - Ensure a "safe" domain
  - $\texttt{take} : (m : \mathbb{N}) \to \texttt{Vec } \alpha\ (m + n) \to \texttt{Vec } \alpha\ m$

Dependently-Typed Programming

- Type checking requires *evaluation* of functions
  - We want Vec Bool (2 + 2) to unify with Vec Bool 4
- Consequence: all functions must be *total*
- Termination checker (heuristics)
  - Structurally-decreasing recursion
  - This passes the check:

$$\begin{aligned} &\text{add} : \mathbb{N} \to \mathbb{N} \to \mathbb{N} \\ &\text{add zero} \quad y = y \\ &\text{add (suc } x') \; y = \text{suc (add } x' \; y) \end{aligned}$$

  - This does not:

$$\begin{aligned} &\text{silly} : \mathbb{N} \to \mathbb{N} \\ &\text{silly zero} \quad = \text{zero} \\ &\text{silly (suc } n') = \text{silly } \lfloor n' /2 \rfloor \end{aligned}$$

Dependently-Typed Programming

- Dependent pattern matching can *rule out* impossible cases

- Classic example: *safe* head function

$$\text{head} : \text{Vec}\ \alpha\ (\text{suc}\ n) \to \alpha$$
$$\text{head}\ (x :: xs) = x$$

  - The **only** constructor returning Vec α (suc n) is _::_

Universiteit Utrecht

Depedent types as logic

- Programming language / Theorem prover
  - Types as propositions, terms as proofs [Wadler, 2014]
- Example:
  - Given the relation:

```
data _≤_ : ℕ → ℕ → Set where
  z≤n : ∀ {n}                → zero  ≤ n
  s≤s : ∀ {m n} → m ≤ n → suc m ≤ suc n
```

  - Proposition:

```
twoLEQFour : 2 ≤ 4
```

  - Proof:

```
twoLEQFour = s≤s (s≤s z≤n)
s≤s (s≤s (z≤n : 0 ≤ 4) : 1 ≤ 4) : 2 ≤ 4
```


Agda

Universiteit Utrecht

Agda syntax for Haskell programmers

- Liberal identifier lexing (Unicode **everywhere**)
  - a≡b+c is a valid identifier, $a \equiv b + c$ an expression
  - Used a lot in Agda's standard library: ×, ⊎, ∧
  - And in Π-Ware: ℂ, ⟦ c ⟧, ⇓, ⇑
- *Mixfix* notation
  - _[_]:=_ is the vector update function: v [# 3] := true.
  - _[_]:=_ v (# 3) true ⟺ v [# 3] := true
- Almost nothing built-in
  - _+_ : ℕ → ℕ → ℕ defined in Data.Nat
  - if_then_else_ : Bool → α → α → α defined in Data.Bool

Agda syntax for Haskell programmers

- Implicit arguments
  - Don't have to be passed if Agda can **guess** it
  - Syntax: $\varepsilon : \{\alpha : \mathsf{Set}\} \to \mathsf{Vec}\ \alpha\ \mathsf{zero}$
- "For all" syntax: $\forall\ n \Longleftrightarrow (n : \_)$
  - Where _ means: guess this type (based on other args)
  - Example:
    - $\forall\ n \to \mathsf{zero} \leq n$
    - $\mathsf{data}\ \_\leq\_ : \mathbb{N} \to \mathbb{N} \to \mathsf{Set}$
- It's common to combine both:
  - $\forall\ \{\alpha\ n\} \to \mathsf{Vec}\ \alpha\ (\mathsf{suc}\ n) \to \alpha \Longleftrightarrow$
    $\{\alpha : \_\}\ \{n : \_\} \to \mathsf{Vec}\ \alpha\ n \to \alpha$

Universiteit Utrecht

Circuit Syntax

Low-level circuits

- Structural representation
- Untyped but *sized*

```
data ℂ′ : ℕ → ℕ → Set
data ℂ′ where
  Nil     : ℂ′ zero zero
  Gate    : (g# : Gates#)  → ℂ′ (|in| g#) (|out| g#)
  Plug    : ∀ {i o}        → (f : Fin o → Fin i) → ℂ′ i o

  DelayLoop : (c : ℂ′ (i + l) (o + l)) {comb′ c} → ℂ′ i o

  _⟫′_   : ℂ′ i m    → ℂ′ m o    → ℂ′ i o
  _|′_   : ℂ′ i₁ o₁  → ℂ′ i₂ o₂  → ℂ′ (i₁ + i₂) (o₁ + o₂)
  _|+′_  : ℂ′ i₁ o   → ℂ′ i₂ o   → ℂ′ (suc (i₁ ⊔ i₂)) o
```


Universiteit Utrecht

Atoms

- How to carry values of an Agda type in *one* wire
- Defined by the Atomic type class in PiWare.Atom

```
record Atomic : Set₁ where
  field
    Atom      : Set
    |Atom|−1  : ℕ
    n→atom    : Fin (suc |Atom|−1) → Atom
    atom→n    : Atom → Fin (suc |Atom|−1)

    inv−left  : ∀ i → atom→n (n→atom i) ≡ i
    inv−right : ∀ a → n→atom (atom→n a) ≡ a

|Atom| = suc |Atom|−1
Atom#  = Fin |Atom|
```


Universiteit Utrecht

Atomic instances

- Examples of types that can be Atomic
  - Bool, `std_logic`, other multi-valued logics
  - Predefined in the library: PiWare.Atom.Bool
- First, define how many atoms we are interested in
  - Need at least 1 (later why)

```
|B|−1 = 1
|B| = suc |B|−1
```

- Friendlier names for the indices (elements of Fin 2)

```
pattern False#  = Fz
pattern True#   = Fs Fz
```


Atomic instance (Bool)

- Bijection between $\{n \in \mathbb{N} \mid n < 2\}$ (Fin 2) and Bool

```
n→B = λ { False# → false;  True#  → true }
B→n = λ { false  → False#; true   → True# }
```

- Proof that n→B and B→n are inverses

```
inv–left–B  = λ { False# → refl; True#  → refl; }
inv–right–B = λ { false  → refl; true   → refl }
```

- With all pieces at hand, we construct the instance

```
Atomic–B = record { Atom      = B
                  ; |Atom|–1  = |B|–1
                  ; n→atom    = n→B
                  ; atom→n    = B→n
                  ; inv–left  = inv–left–B
                  ; inv–right = inv–right–B }
```


Gates

- Circuits parameterized by collection of *fundamental gates*
- Examples:
  - {NOT, AND, OR} (BoolTrio)
  - {NAND}
  - Arithmetic, Crypto, etc.
- The definition of what means to be such a collection is in PiWare.Gates.Gates

The Gates type class

```
W : ℕ → Set
W = Vec Atom
record Gates : Set where
  field
    |Gates|     : ℕ
    |in| |out|  : Fin |Gates| → ℕ
    spec        : (g : Fin |Gates|)
                  → (W (|in| g) → W (|out| g))

  Gates#  = Fin |Gates|
```


Gates instances

- Example: PiWare.Gates.BoolTrio
- First, how many gates are there in the library

  $|\mathsf{BoolTrio}| = 5$

- Then the friendlier names for the indices

```
pattern FalseConst#  = Fz
pattern TrueConst#   = Fs Fz
pattern Not#         = Fs (Fs Fz)
pattern And#         = Fs (Fs (Fs Fz))
pattern Or#          = Fs (Fs (Fs (Fs Fz)))
```


Gates instance (BoolTrio)

- Defining the *interfaces* of the gates

```
|in| FalseConst#  = 0
|in| TrueConst#   = 0
|in| Not#         = 1
|in| And#         = 2
|in| Or#          = 2

|out| _ = 1
```

- And the specification function for each gate

```
spec-false  _            = [ false   ]
spec-true   _            = [ true    ]
spec-not    (x :: ε)     = [ not x   ]
spec-and    (x :: y :: ε) = [ x ∧ y   ]
spec-or     (x :: y :: ε) = [ x ∨ y   ]
```


Universiteit Utrecht

Gates instance (BoolTrio)

- Mapping each gate index to its respective specification

```
specs–BoolTrio FalseConst#  = spec–false
specs–BoolTrio TrueConst#   = spec–true
specs–BoolTrio Not#         = spec–not
specs–BoolTrio And#         = spec–and
specs–BoolTrio Or#          = spec–or
```

- With all pieces at hand, we construct the instance

```
BoolTrio : Gates
BoolTrio = record  { |Gates|  = |BoolTrio|
                   ; |in|     = |in|
                   ; |out|    = |out|
                   ; spec     = specs–BoolTrio }
```


High-level circuits

- User is not supposed to describe circuits at low level ($\mathbb{C}'$)
- The high level circuit type ($\mathbb{C}$) allows for *typed* circuit interfaces
  - Input and output indices are Agda types

$$\begin{aligned}&\mathsf{data}\ \mathbb{C}\ (\alpha\ \beta : \mathsf{Set})\ \{i\ j : \mathbb{N}\} : \mathsf{Set}\ \mathsf{where}\\ &\quad \mathsf{Mk}\mathbb{C} : \llbracket\ s\alpha : \Downarrow \mathsf{W} \Uparrow\ \alpha\ \{i\}\ \rrbracket\ \llbracket\ s\beta : \Downarrow \mathsf{W} \Uparrow\ \beta\ \{j\}\ \rrbracket\\ &\qquad\qquad \to \mathbb{C}'\ i\ j \to \mathbb{C}\ \alpha\ \beta\ \{i\}\ \{j\}\end{aligned}$$

- $\mathsf{Mk}\mathbb{C}$ takes:
  - Low level description ($\mathbb{C}'$)
  - Information on how to *synthesize* elements of α and β
    - Passed as *instance arguments* (class constraints)

Synthesizable

- ⇓W⇑ type class (pronounced `Synthesizable`)
  - Describes how to synthesize a given Agda type (α)
  - Two fields: from element of α to a *word* and back

```
record ⇓W⇑ (α : Set) {i : ℕ} : Set where
  constructor ⇓W⇑[_,_]
  field
    ⇓ : α → W i
    ⇑ : W i → α
```


Universiteit Utrecht

⇓W⇑ instances

- Any *finite* type can have such an instance
- Predefined in the library: Bool; _×_; _⊎_; Vec
- Example: instance for products (_×_)

$$
\begin{aligned}
&\Downarrow\mathsf{W}\Uparrow\!-\!\times : \{\!|\ s\alpha : \Downarrow\mathsf{W}\Uparrow\ \alpha\ \{i\}\ |\!\}\ \{\!|\ s\beta : \Downarrow\mathsf{W}\Uparrow\ \beta\ \{j\}\ |\!\} \\
&\qquad\qquad \rightarrow \Downarrow\mathsf{W}\Uparrow\ (\alpha \times \beta) \\
&\Downarrow\mathsf{W}\Uparrow\!-\!\times\ \{\!|\ s\alpha\ |\!\}\ \{\!|\ s\beta\ |\!\} = \Downarrow\mathsf{W}\Uparrow[\ \mathsf{down}\ ,\ \mathsf{up}\ ] \\
&\quad \textsf{where}\ \ \mathsf{down} : (\alpha \times \beta) \rightarrow \mathsf{W}\ (i + j) \\
&\qquad\qquad \mathsf{down}\ (a\ ,\ b) = (\Downarrow\ a)\ +\!+\ (\Downarrow\ b) \\
&\qquad\qquad \mathsf{up} : \mathsf{W}\ (i + j) \rightarrow (\alpha \times \beta) \\
&\qquad\qquad \mathsf{up}\ w\ \textsf{with}\ \mathsf{splitAt}\ i\ w \\
&\qquad\qquad \mathsf{up}\ .(\Downarrow a\ +\!+\ \Downarrow b)\ |\ \Downarrow a\ ,\ \Downarrow b\ ,\ \mathsf{refl} = \Uparrow\ \Downarrow a\ ,\ \Uparrow\ \Downarrow b
\end{aligned}
$$

Universiteit Utrecht

Synthesizable

- Both fields $\Downarrow$ and $\Uparrow$ should be inverses of each other
  - Due to how high-level simulation is defined using $\Downarrow$ and $\Uparrow$
- Not enforced as a field of of $\Downarrow \mathbb{W} \Uparrow$
  - Too big of a proof burden while quick prototyping

Semantics

Circuit semantics

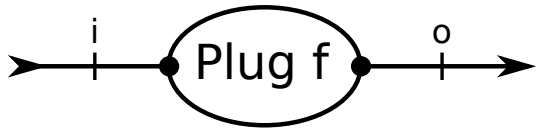
- *Synthesis* semantics: produce a *netlist*
  - Tool integration / implement in FPGA or ASIC.
- *Simulation* semantics: *execute* a circuit
  - Given circuit model and inputs, calculate outputs
- Other semantics possible:
  - Timing analysis, power estimation, etc.
  - This possibility guided Π-Ware's development

Synthesis semantics

- Netlist: digraph with *gates* as nodes and *buses* as edges
- Synthesis semantics: given netlists of subcircuits, build combination

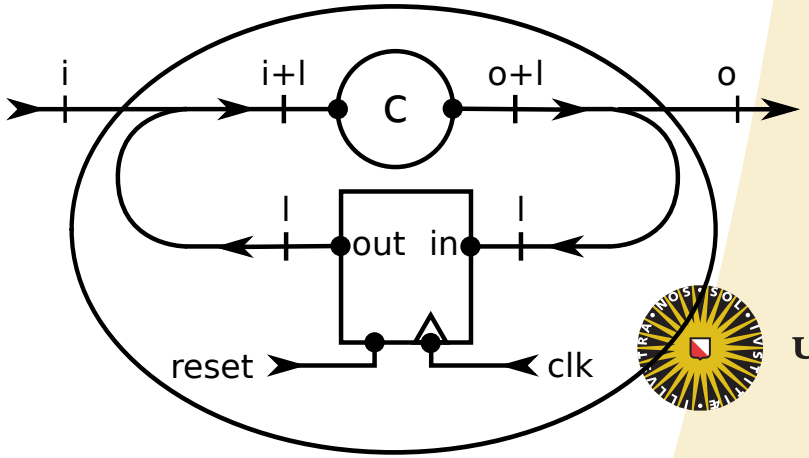
$$\mathsf{Nil} : \mathbb{C}\ 0\ 0$$

$$\frac{\mathsf{i\ o} : \mathbb{N} \qquad \mathsf{f} : \mathsf{Fin\ o} \to \mathsf{Fin\ i}}{\mathsf{Plug\ f} : \mathbb{C}\ \mathsf{i\ o}}$$



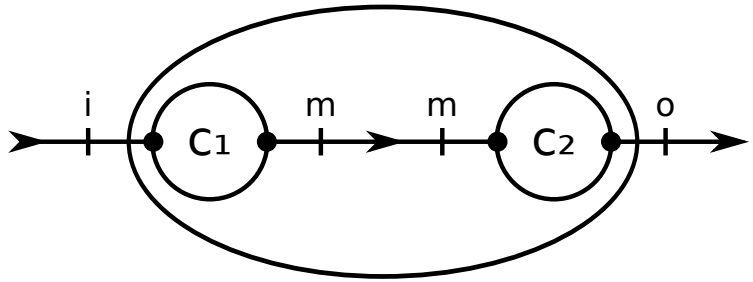
$$\frac{\mathsf{g\#} : \mathsf{Gate\#}}{\mathsf{Gate\ g\#} : \mathbb{C}\ (\mathsf{ins\ g\#})\ (\mathsf{outs\ \ g\#})}$$

$$\frac{\mathsf{c} : \mathbb{C}\ (\mathsf{i+l})\ (\mathsf{o+l})}{\mathsf{DelayLoop} : \mathbb{C}\ \mathsf{i\ o}}$$

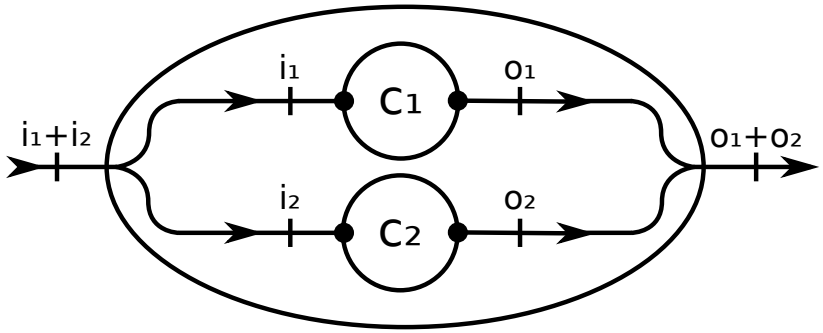


Synthesis semantics

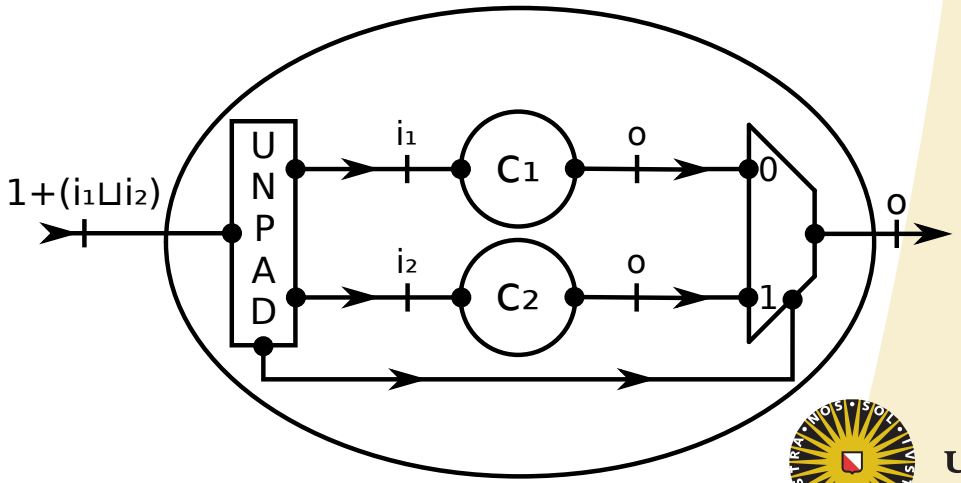
$$\frac{c_1 : \mathbb{C}\ i\ m \qquad c_2 : \mathbb{C}\ m\ o}{c_1 \gg' c_2 : \mathbb{C}\ i\ o}$$



$$\frac{c_1 : \mathbb{C}\ i_1\ o_1 \qquad c_2 : \mathbb{C}\ i_2\ o_2}{c_1 \mid' c_2 : \mathbb{C}\ (i_1{+}i_2)\ (o_1{+}o_2)}$$



$$\frac{c_1 : \mathbb{C}\ i_1\ o \qquad c_2 : \mathbb{C}\ i_2\ o}{c_1 \mid{+}' c_2 : \mathbb{C}\ (1{+}(i_1 \sqcup i_2))\ o}$$



Universiteit Utrecht

Synthesis semantics

Missing "pieces":

- Adapt Atomic
  - New field: a VHDLTypeDecl
    - Such as: `type ident is (elem1, elem2);`
    - Enumerations, integers (ranges), records.
  - New field: atomVHDL : Atom# → VHDLExpr
- Adapt Gates
  - For each gate, a corresponding VHDLEntity
  - netlist : $(g\# : \text{Gates\#}) \rightarrow \text{VHDLEntity}\ (|\text{in}|\ g\#)\ (|\text{out}|\ g\#)$
    - The VHDL entity has the *interface* of corresponding gate

Simulation semantics

- Two levels of abstraction
  - High-level simulation ($[\![\_]\!]$) for high-level circuits ($\mathbb{C}$)
  - Low-level simulation ($[\![\_]\!]'$) for low-level circuits ($\mathbb{C}'$)
- Two kinds of simulation
  - Combinational simulation ($[\![\_]\!]$) for stateless circuits
  - Sequential simulation ($[\![\_]\!]*$) for stateful circuits
- High level defined in terms of low level

$$[\![\_]\!] : \forall \{\alpha\ i\ \beta\ j\} \to (c : \mathbb{C}\ \alpha\ \beta\ \{i\}\ \{j\}) \to (\alpha \to \beta)$$
$$[\![\ \mathsf{Mk}\mathbb{C}\ \{\!\{\ s\alpha\ \}\!\}\ \{\!\{\ s\beta\ \}\!\}\ c'\ ]\!] = \Uparrow \circ [\![\ c'\ ]\!]' \circ \Downarrow$$

Universiteit Utrecht

Combinational simulation (excerpt)

$$[\![\_]\!]' : \forall\ \{i\ o\} \to (c : \mathbb{C}'\ i\ o)\ \{p : \mathsf{comb}'\ c\} \to (\mathsf{W}\ i \to \mathsf{W}\ o)$$
$$[\![\ \mathsf{Nil}\ ]\!]' = \mathsf{const}\ \varepsilon$$
$$[\![\ \mathsf{Gate}\ g\#\ ]\!]' = \mathsf{spec}\ g\#$$
$$[\![\ \mathsf{Plug}\ p\ ]\!]' = \mathsf{plugOutputs}\ p$$

$$[\![\ \mathsf{DelayLoop}\ c\ ]\!]'\ \{()\}\ v$$

$$[\![\ c_1\ \gg'\ c_2\ ]\!]'\ \{p_1\ ,\ p_2\} = [\![\ c_2\ ]\!]'\ \{p_2\} \circ [\![\ c_1\ ]\!]'\ \{p_1\}$$

$$[\![\ \_|+'\_\ \{i_1\}\ c_1\ c_2\ ]\!]'\ \{p_1\ ,\ p_2\} =$$
$$[\ [\![\ c_1\ ]\!]'\ \{p_1\}\ ,\ [\![\ c_2\ ]\!]'\ \{p_2\}\ ]' \circ \mathsf{untag}\ \{i_1\}$$

- Remarks:
  - Proof requires c to be combinational
  - Gate case uses specification function
  - DelayLoop case can be *discharged*

Sequential simulation

- Inputs and outputs become Streams
  - $\mathbb{C}'\ i\ o \Longrightarrow \mathsf{Stream}\ (\mathsf{W}\ i) \to \mathsf{Stream}\ (\mathsf{W}\ o)$
  - Stream: infinite list
- We can't write a recursive evaluation function over Streams
  - *Sum* case $(\_|+'\_)$ needs a function of type $(\mathsf{Stream}\ (\alpha \uplus \beta) \to \mathsf{Stream}\ \alpha \times \mathsf{Stream}\ \beta)$
    - What if there are no *lefts* (or *rights*)?
- A stream function is not an accurate model for hardware
  - A function of type $(\mathsf{Stream}\ \alpha \to \mathsf{Stream}\ \beta)$ can "look ahead"
  - For example, $\mathsf{tail}\ (x_0 :: x_1 :: x_2 :: xs) = x_1 :: x_2 :: xs$

Universiteit Utrecht

Causal stream functions

Solution: sequential simulation based on *causal* stream function

Some definitions:

- Causal context: past + present values

$$\Gamma\mathsf{c} : (\alpha : \mathsf{Set}) \to \mathsf{Set}$$
$$\Gamma\mathsf{c}\ \alpha = \alpha \times \mathsf{List}\ \alpha$$

- Causal stream function: produces **one** (current) output

$$\_\Rightarrow\mathsf{c}\_ : (\alpha\ \beta : \mathsf{Set}) \to \mathsf{Set}$$
$$\alpha \Rightarrow\mathsf{c}\ \beta = \Gamma\mathsf{c}\ \alpha \to \beta$$

Universiteit Utrecht

Causal sequential simulation

- Core sequential simulation function:

$$
\begin{aligned}
&[\![\_]\!]\mathsf{c} : \{i\ o : \mathbb{N}\} \to \mathbb{C}'\ i\ o \to (\mathsf{W}\ i \Rightarrow\mathsf{c}\ \mathsf{W}\ o)\\
&[\![\ \mathsf{Nil}\ ]\!]\mathsf{c}\ (w^0\ , \_) = [\![\ \mathsf{Nil}\ ]\!'\ w^0\\
&[\![\ \mathsf{Gate}\ g\#\ ]\!]\mathsf{c}\ (w^0\ , \_) = [\![\ \mathsf{Gate}\ g\#\ ]\!'\ w^0\\
&[\![\ \mathsf{Plug}\ p\ ]\!]\mathsf{c}\ (w^0\ , \_) = \mathsf{plugOutputs}\ p\ w^0\\
&[\![\ \mathsf{DelayLoop}\ c\ \{p\}\ ]\!]\mathsf{c} = \mathsf{take_v}\ j \circ \mathsf{delay}\ c\ \{p\}
\end{aligned}
$$

$$
[\![\ c_1 \gg' c_2\ ]\!]\mathsf{c} = [\![\ c_2\ ]\!]\mathsf{c} \circ \mathsf{map}^+\ [\![\ c_1\ ]\!]\mathsf{c} \circ \mathsf{tails}^+
$$

- Nil, Gate and Plug cases use combinational simulation
- DelayLoop calls a recursive helper (delay)
- Example structural case: $\_\gg'\_$ (sequence)
  - Context of $[\![\ c_1\ ]\!]\mathsf{c}$ is context of the whole compound
  - Context of $[\![\ c_2\ ]\!]\mathsf{c}$ is past and present *outputs* of c1

Sequential simulation

- We can then "run" the step-by-step function to produce a whole Stream
  - Idea from "The Essence of Dataflow Programming" [Uustalu and Vene, 2005]

$$\begin{aligned}
&\mathsf{runc'} : (\alpha \Rightarrow \mathsf{c}\ \beta) \to (\Gamma\mathsf{c}\ \alpha \times \mathsf{Stream}\ \alpha) \to \mathsf{Stream}\ \beta \\
&\mathsf{runc'}\ f\ ((x^0\ ,\ x^-)\ ,\ (x^1\ ::\ x^+)) = \\
&\quad f\ (x^0\ ,\ x^-)\ ::\ \sharp\ \mathsf{runc'}\ f\ ((x^1\ ,\ x^0\ ::\ x^-)\ ,\ \flat\ x^+)
\end{aligned}$$

$$\begin{aligned}
&\mathsf{runc} : (\alpha \Rightarrow \mathsf{c}\ \beta) \to (\mathsf{Stream}\ \alpha \to \mathsf{Stream}\ \beta) \\
&\mathsf{runc}\ f\ (x^0\ ::\ x^+) = \mathsf{runc'}\ f\ ((x^0\ ,\ [])\ ,\ \flat\ x^+)
\end{aligned}$$

- Obtaining the stream-based simulation function:

$$\begin{aligned}
&[\![_]\!]\mathsf{*'} : \forall\ \{i\ o\} \to \mathbb{C}'\ i\ o \to (\mathsf{Stream}\ (\mathsf{W}\ i) \to \mathsf{Stream}\ (\mathsf{W}\ o)) \\
&[\![_]\!]\mathsf{*'} = \mathsf{runc} \circ [\![_]\!]\mathsf{c}
\end{aligned}$$

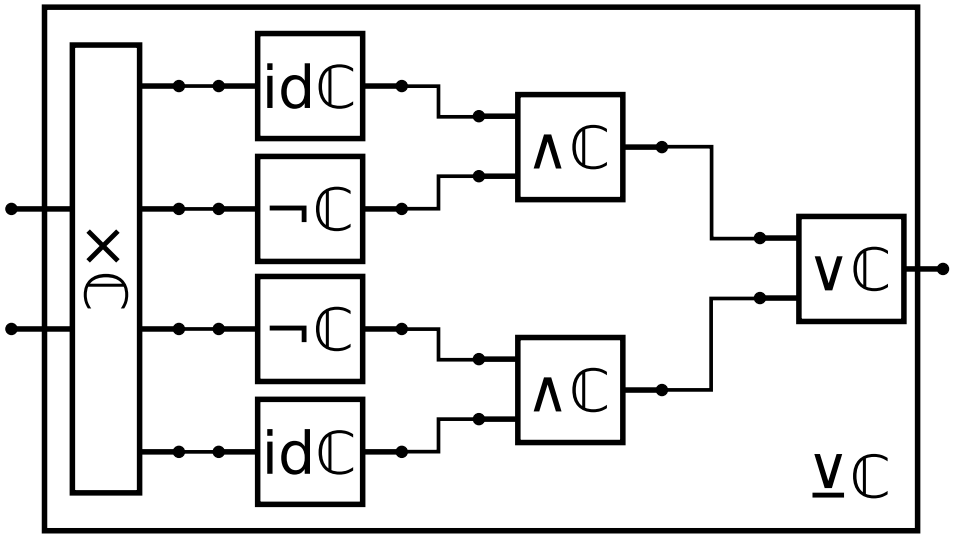
Universiteit Utrecht

Proofs

Properties of circuits

- Tests and proofs about circuits depend on the *semantics*
  - We focused on the functional simulation semantics
  - Other possibilities (gate count, critical path, etc.)
- Very simple sample circuit to illustrate: `XOR`

Sample circuit: XOR



$$\underline{\vee}\mathbb{C} : \mathbb{C}\ (B \times B)\ B$$

$$\begin{aligned}\underline{\vee}\mathbb{C} = \ & \mathsf{pFork}\times \\ & \gg (\neg\mathbb{C} \parallel \mathsf{id}\mathbb{C} \gg \wedge\mathbb{C}) \parallel (\mathsf{id}\mathbb{C} \parallel \neg\mathbb{C} \gg \wedge\mathbb{C}) \\ & \gg \vee\mathbb{C}\end{aligned}$$

Universiteit Utrecht

Specification of XOR

- To define *correctness* we need a *specification function*
  - Listing all possibilities (truth table)
  - Based on pre-exisiting functions (standard library)
- Truth table

$$
\begin{aligned}
&\veebar\mathbb{C}\text{–spec–table} : (\mathsf{B} \times \mathsf{B}) \to \mathsf{B} \\
&\veebar\mathbb{C}\text{–spec–table}\ (\mathsf{false}\ ,\ \mathsf{false}) = \mathsf{false} \\
&\veebar\mathbb{C}\text{–spec–table}\ (\mathsf{false}\ ,\ \mathsf{true}\ ) = \mathsf{true} \\
&\veebar\mathbb{C}\text{–spec–table}\ (\mathsf{true}\ ,\ \mathsf{false}) = \mathsf{true} \\
&\veebar\mathbb{C}\text{–spec–table}\ (\mathsf{true}\ ,\ \mathsf{true}\ ) = \mathsf{false}
\end{aligned}
$$

Proof of XOR (truth table)

```
⊻ℂ−proof−table : ⟦ ⊻ℂ ⟧ (a , b) ≡ ⊻ℂ−spec−table (a , b)
⊻ℂ−proof−table false false = refl
⊻ℂ−proof−table false true  = refl
⊻ℂ−proof−table true  false = refl
⊻ℂ−proof−table true  true  = refl
```

- Proof by *case analysis*
  - Can probably be automated by *reflection* [van der Walt and Swierstra, 2013]

Specification of XOR

- Based (_xor_) from Data.Bool

```
_xor_ : B → B → B
true  xor b = not b
false xor b = b
```

- Adapted interface to match exactly ⅄ℂ

```
⅄ℂ-spec-subfunc : (B × B) → B
⅄ℂ-spec-subfunc = uncurry′ _xor_
```


Proof of XOR (pre-existing)

- Proof based on ∨ℂ−spec−subfunc

  ∨ℂ−proof−subfunc : ⟦ ∨ℂ ⟧ (a , b) ≡ ∨ℂ−spec−subfunc (a , b)
  ∨ℂ−proof−subfunc = ∨ℂ−xor−equiv

- Need a lemma to complete the proof
  - Circuit is defined using {NOT, AND, OR}
  - _xor_ is defined directly by pattern matching

∨ℂ−xor−equiv : (not a ∧ b) ∨ (a ∧ not b) ≡ (a xor b)

Circuit "families"

- We can also prove properties of circuit "families"
- Example: an `AND` gate definition with generic number of inputs

$$
\begin{aligned}
&\mathsf{andN'} : \forall\ n \to \mathbb{C}'\ n\ 1 \\
&\mathsf{andN'}\ \mathsf{zero} = \mathsf{T}\mathbb{C}' \\
&\mathsf{andN'}\ (\mathsf{suc}\ n) = \mathsf{id}\mathbb{C}'\ |'\ \mathsf{andN'}\ n\ \rangle\!\rangle'\ \wedge\mathbb{C}'
\end{aligned}
$$

- Example proof: when all inputs are true, output is true
  - For *any* number of inputs
  - Proof by induction on n (number of inputs)

Problems

- This proof is done at the *low level*

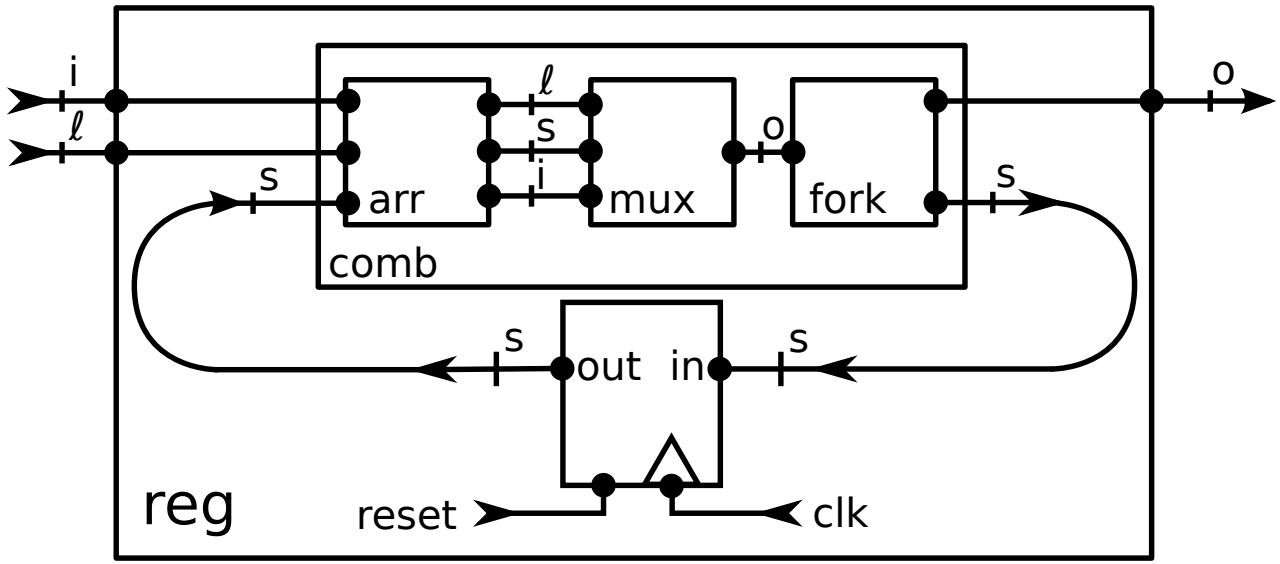
```
proof−andN′ : ∀ n → ⟦ andN′ n ⟧′ (replicate true) ≡ [ true ]
proof−andN′ zero    = refl
proof−andN′ (suc n) = cong (spec−and ∘ (_::_ true))
                           (proof−andN′ n)
```

- Still problems with inductive proofs in the high level
  - Guess: definition of $\mathbb{C}$ and $[\![\_]\!]$ prevent goal reduction

Universiteit Utrecht

Sequential proofs

- Example of sequential circuit: a *register*



- Respective Π-Ware circuit description

```
reg : ℂ (B × B) B
reg = delayℂ (arr ⟫ mux2to1 ⟫ ×ℂ)
    where arr = (⇅ℂ || idℂ) ⟫ ALRℂ ⟫ (idℂ || ⇅ℂ)
```


Universiteit Utrecht

Register example

- Example (test case) of register behaviour

```
loads inputs : Stream Bool
loads  = true :: ♯ (true  :: ♯ (false :: ♯ repeat false))
inputs = true :: ♯ (false :: ♯ (true  :: ♯ repeat false))

actual = take 42 (⟦ reg ⟧* $ zipWith _,_ inputs loads)

test−reg = actual ≡ true ◁ false ◁ replicate false
```

- Still problems with *infinite* expected vs. actual comparisons
  - Normal Agda equality (_≡_) does not work
  - Need to use *bisimilarity*

Summary

Universiteit Utrecht

What Π-Ware achieves

- Several design activities in the *same language*
  - Description (untyped / typed)
  - Simulation
  - Synthesis
  - Verification (inductive families of circuits)
- Well-typed descriptions ($\mathbb{C}$) at *compile time*
  - Low-level descriptions ($\mathbb{C}'$) / netlists are *well-sized*
- Type safety and totality of simulation due to Agda

Current limitations / trade-offs

- Interface of generated netlists is always *flat*
  - One input, one output

```
entity fullAdd8 is
port (
    inputs  : in  std_logic_vector(16 downto 0);
    outputs : out std_logic_vector(8 downto 0)
);
end fullAdd8;
```

- Due to the indices of $\mathbb{C}'$ (naturals)
  - Can't distinguish $\mathbb{C}'\ (1 + 8 + 8)\ (8 + 1)$ from $\mathbb{C}'\ 17\ 9$

Current limitations / trade-offs

- Proofs for high-level families of circuits
  - Probably due to definitions of $\mathbb{C}$ and $[\![\_]\!]$
- Proofs with infinite comparisons (sequential circuits)

Future work

Universiteit Utrecht

Future work

- Automatic proof by reflection for finite cases
- Prove properties of combinators in Agda
  - Algebraic properties
- Automatic generation of ⇓W⇑ (`Synthesizable`) instances
- More (higher) layers of abstraction

Universiteit Utrecht

Thank you!

Questions?

Mede mogelijk gemaakt door:

Utrechts Universiteitsfonds

Universiteit Utrecht

References I

Bjesse, P., Claessen, K., Sheeran, M., and Singh, S. (1998).

Lava: hardware design in Haskell.
SIGPLAN Not., 34(1):174–184.

Launchbury, J., Lewis, J. R., and Cook, B. (1999).
On embedding a microarchitectural design language within haskell.
SIGPLAN Not., 34(9):60–69.

Oury, N. and Swierstra, W. (2008).
The power of pi.
SIGPLAN Not., 43(9):39–50.

Universiteit Utrecht

References II

Sander, I. and Jantsch, A. (1999).
Formal system design based on the synchrony hypothesis, functional models, and skeletons.
In *VLSI Design, 1999. Proceedings. Twelfth International Conference On*, pages 318–323. IEEE.

Sheeran, M. (1984).
MuFP, a language for VLSI design.
In *Proceedings of the 1984 ACM Symposium on LISP and Functional Programming*, LFP '84, pages 104–112, New York, NY, USA. ACM.

Uustalu, T. and Vene, V. (2005).
The essence of dataflow programming.
In *Proceedings of the Third Asian Conference on Programming Languages and Systems*, APLAS'05, pages 2–18, Berlin, Heidelberg. Springer-Verlag.

Universiteit Utrecht

References III

van der Walt, P. and Swierstra, W. (2013).
Engineering proof by reflection in Agda.
In *Implementation and Application of Functional Languages*, pages 157–173. Springer.

Wadler, P. (2014).
Propositions as types.
Unpublished note, `http://homepages.inf.ed.ac.uk/wadler/papers/propositions-as-types/propositions-as-types.pdf`.

Universiteit Utrecht